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Poison Distribution Resulting in a Desired Power Distribution

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ABSTRACT

The problem of determination of the poison distribution (control rods, burnable poison) in a reactor which results in a desired power distribution is formulated and solved as a linear programming problem. For realistic reactor problems (two or three dimensions, more than one neutron group) the resulting LP problem is very large and its solution by known LP routines is inefficient. To avoid this difficulty a method is developed which calculates all the required input to start the LP algorithm from solutions of the neutronic equations.

The output of the LP algorithm in addition to the poison distribution gives the corresponding flux distribution.

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INTRODUCTION

The total power output of a reactor of given size depends on the maximum allowable power density and the power distribution. The thermal and mechanical reactor designs are also based on a given power distribution. Fuel depletion during the course of reactor operation causes the power distribution to change. This change can be controlled, within certain design limits, by application of proper poison management procedures (control rod withdrawal and insertion patterns, burnable poison distribution, etc.). Therefore, if a desired power distribution has been specified, a calculational method is required to determine the initial poison distribution and its variation during reactor operation so that the desired power distribution is realized.

In usual depletion calculations after each depletion step the search for the control rod distribution which satisfies reactor criticality and a desired power distribution is done iteratively in a trial and error fashion.[1] This places a heavy computing burden on the conventional depletion calculations.

Crowther[1] developed the Power-Control method for solution of steady-state neutron diffusion-theory problems for BWR's in which the material nuclear properties depend strongly on the power distribution. The Power-Control method is also an iterative method. In a two neutron group model, it starts with a desired initial power distribution and determines a poison distribution which satisfies reactor criticality and the desired power distribution. But constraints such as the absorber concentration be non-

negative, less than a certain magnitude and be non-continuously distributed but in certain finite steps, may not be satisfied by the determined poison distribution. To satisfy these constraints an iterative algorithm is applied. The determined poison distribution is changed so that the constraints are satisfied. From the revised poison distribution a new power distribution is calculated and reactor criticality is checked. If the reactor is not critical the new power distribution is used to calculate another poison distribution satisfying the new power distribution and reactor criticality. The iterations are repeated until a poison distribution is determined which satisfies the required physical constraints, reactor criticality and a power distribution close to the desired distribution.

Turney and Fenech[2] used dynamic programming and a direct flux-synthesis method to determine the control rod program which minimizes power peaking throughout the life of a PWR. Dynamic programming gives a systematic approach which discards nonoptimal paths as soon as possible and leads to a significant reduction in the number of paths to be considered over those resulting by a straight factorial approach of all possible options. But even though, for the problem of interest the number of paths to be considered is large. Turney and Fenech to reduce this number considered only two control banks and imposed the constraint that the control rods must always move only in three prescribed manners, namely: a) moving only the outer bank; b) moving both the same; and c) moving only the inner bank but never allowing it to be further out than the outer bank.

Workers of the Halden Reactor Project[3] have formulated in terms of quadratic programming the problem of determination of the control rod positions which minimize a weighted sum of squares of deviations between pre-

dicted and desired power densities and they have developed an algorithm for the solution of the problem. The method is proposed for on-line use during reactor operation and the required parameters are directly obtained from the reactor by on-line identification techniques.

In this study, the problem of determination of the poison distribution (control rods, burnable poison) in a reactor which results in a desired power distribution is formulated and solved as a linear programming problem. For realistic reactor problems (two or three dimensions, more than one neutron group) the resulting LP problem is very large and its solution by known LP routines requires excessive computer time and gives rise to prohibitive roundoff errors. To overcome this difficulty a more efficient method than the known methods of the LP routines is developed which calculates all the required input to start the LP algorithm from solutions of the neutronic equations (multigroup diffusion equations, transport theory equations, etc.).

The present method by using LP does not rely on trial and error, physical constraints on the poison concentration are directly treated, it does not have the dimensionality problem of dynamic programming, which arises from the large number of paths required to be examined for a realistic problem, and has all the computational advantages of LP compared to other optimization techniques. For the neutronic discription many neutron groups can be used.

Numerical applications of the method are presented for a fast reactor of infinite cylindrical geometry.

THEORY

I. Formulation of the Problem

In the context of multigroup diffusion theory, the i'th neutron group flux $\phi_i(\bar{r})$ and the power density $W(\bar{r})$ are given by the relations

$$\nabla\cdot D_i(\bar{r})\nabla\phi_i(\bar{r}) - \Sigma_{a,i}(\bar{r})\phi_i(\bar{r}) - \sum_{h=i+1}^{N} \Sigma_{(i\to h)}(\bar{r})\phi_i(\bar{r})$$

$$+ \sum_{h=1}^{i-1} \Sigma_{(h\to i)}(\vec{r})\phi_h(\bar{r}) + \frac{\chi_i}{k} \sum_{h=1}^{N} \nu_h \Sigma_{f,h}(\bar{r})\phi_h(\bar{r})$$

$$- N_p(\bar{r})\sigma^P_{\alpha,i}\,\phi_i(\bar{r}) = 0 \qquad (i = 1,\ldots,N) \tag{1}$$

$$W(\bar{r}) = \sum_{h=1}^{N} \Sigma_{f,h}(\bar{r})\phi_h(\bar{r}) \tag{2}$$

where

$D_i(\bar{r})$ = diffusion coefficient for group i

$\Sigma_{a,i}(\bar{r})$ = macroscopic absorption cross section for group i (the poison absorption cross section is not included)

$\Sigma_{(i\to h)}(\bar{r})$ = macroscopic downscattering cross section for transfer from group i to group h by elastic and inelastic scattering

$k = k_{eff}$

χ_i = fraction of neutrons born into group i

$\Sigma_{f,h}(\bar{r})$ = macroscopic fission cross section for group h

ν_h = number of neutrons released per fission induced by neutrons of the h'th group

$N_p(\bar{r})$ = poison concentration

$\sigma^{P}_{a,i}$ = microscopic absorption cross section of poison material for group i

N = number of neutron groups

In Eqs. (1) it has been assumed that absorption is the only important neutron interaction with poison. But other interactions can also be included.

If the poison distribution is perturbed by $\delta N_p(\bar{r})$ around an operating distribution $N^o_p(\bar{r})$, and if the perturbation is small, second order terms can be neglected and Eqs. (1) are written as

$$\nabla\cdot D_i(\bar{r})\nabla\phi_i(\bar{r}) - \Sigma_{a,i}(\bar{r})\phi_i(\bar{r}) - \sum_{h=i+1}^{N} \Sigma_{(i\to h)}(\bar{r})\phi_i(\bar{r})$$

$$+ \sum_{h=1}^{i-1} \Sigma_{(h\to i)}(\bar{r})\phi_h(\bar{r}) + \frac{\chi_i}{k} \sum_{h=1}^{N} \nu_h\Sigma_{f,h}(\bar{r})\phi_h(\bar{r})$$

$$- N^o_p(\bar{r})\sigma^P_{a,i}\phi_i(\bar{r}) - \delta N_p(\bar{r})\sigma^P_{a,i}\phi^o_i(\bar{r}) = 0 \quad (z = 1,\ldots,N) \qquad (3)$$

The superscript "o" is used to denote quantities evaluated at the operating poison distribution $N^o_p(\bar{r})$.

If the reactor is critical, k = 1, at the operating distribution $N^o_p(\bar{r})$, to remain critical after the perturbation, the distribution $\delta N_p(\bar{r})$ must satisfy a criticality condition. Such a condition is given by perturbation theory in the form

$$\int_V \delta N_p(\bar{r}) \sum_{i=1}^{N} \sigma^P_{a,i}\phi^o_i(\bar{r})\psi^o_i(\bar{r})d\bar{r} = 0 \qquad (4)$$

where

V = reactor volume

ψ_i = adjoint flux for group i

Eqs. (2), (3) and (4) can be written as a set of algebraic equations by using an appropriate method for this, for example by using the finite difference technique for differentiation and the trapezoidal rule for integration. This set of algebraic equations is written as

$$\begin{aligned} \underline{L}\,\underline{\phi} + \underline{\beta}_{\ell}\,\delta N_{p,\ell} + \underline{M}\,\underline{\delta N^*_p} &= \underline{0} \\ \mathrm{Pert}_{\ell}\,\delta N_{p,\ell} + \underline{\mathrm{Pert}}\,\underline{\delta N^*_p} &= 0 \\ \underline{P}\,\underline{\phi} + \underline{E}\,\underline{S} &= \underline{W}_D \end{aligned} \tag{5}$$

where

$\underline{L}$ = the matrix which results from the terms of Eqs. (3) containing the flux vector $\underline{\phi}$

$\delta N_{p,\ell}$ = the ℓ'th component of the poison distribution vector

$\underline{\delta N_p}$, $\underline{\delta N_p} = [\delta N_{p,\ell}, \underline{\delta N^*_p}]$

$\underline{\beta}_{\ell}$ = the vector which results from the last term of Eqs. (3) containing the component $\delta N_{p,\ell}$

$\underline{M}$ = the matrix which results from the last term of Eqs. (3) containing the components of vector $\underline{\delta N^*_p}$

Pert_{ℓ}, $\underline{\mathrm{Pert}}$ = quantities resulting from Eq. (4)

$\underline{P}$ = the matrix which results from Eq. (2)

$\underline{W}_D$ = the desired power distribution vector

$\underline{S}$ = a positive slack vector resulting from the difference of the desired power distribution and the existing power distribution

$\underline{E}$ = a matrix whose elements are zero or one and their signs are defined such that vector $\underline{S}$ is positive

The poison distribution vector $\underline{\delta N}_p$ has been split into two main components, $\delta N_{p,\ell}$ and $\underline{\delta N^*}_p$ in Eqs. (5) for reasons which will become apparent later. The vector $\underline{\phi}$ in Eqs. (5) is normalized such that for any vector $\underline{\delta N}_p$ one of the components of the vector $\underline{S}$, and always the same, has the value zero.

In the subsequent discussion, it is assumed that the reader is familiar with the concepts, theorems, and terminology of linear programming discussed in standard texts (e.g., Ref. 4).

In terms of Eqs. (5), the problem of determination of the poison distribution which results in the desired power distribution is stated as follows. Determine a vector $\underline{\delta N}_p$ such that Eqs. (5) are satisfied with all the components of the vector $\underline{S}$ equal to zero.

If the vector $\underline{S}$ is positive, the requirement to make all its components equal to zero is equivalent to minimizing the quantity

$$Z = \sum_i S_i \qquad (6)$$

where S_i is the i'th component of the vector $\underline{S}$. Since relations (5) and (6) are linear, the just cited problem can be stated as a linear programming problem as follows: Determine the vector $\underline{\delta N}_p$ such that Eqs. (5) are satisfied and

$$Z = \sum_i S_i = \text{Minimum} = 0 \quad .$$

Therefore linear programming can be used if it can be assured that $\underline{S}$ remains always positive. But if $\underline{S}$ is positive in the initial basic feasible solution the LP algorithm keeps it positive at the following iteration steps. Therefore it is needed to assure only that $\underline{S}$ is positive in the initial basic feasible solution. This is done easily by defining the signs of the elements of matrix $\underline{E}$ such that $\underline{S}$ is positive in the initial basic feasible solution.

Relations (5) and (6) can be written in a more compact form as

$$\underline{A}\,\underline{x} = \chi_1 \underline{a}_1 + \chi_2 \underline{a}_2 + \cdots + \chi_n \underline{a}_n = \underline{b} \tag{7}$$

$$Z = \underline{C}\,\underline{x} \tag{8}$$

where

$$\underline{A} = \begin{bmatrix} \underline{L} & \underline{\beta}_\ell & \underline{M} & \underline{0} \\ 0 & \text{Pert}_\ell & \underline{\text{Pert}} & 0 \\ \underline{P} & \underline{0} & \underline{0} & \underline{E} \end{bmatrix} \tag{9a}$$

$$\underline{x} = [\underline{\phi},\ \delta N_{p,\ell},\ \underline{\delta N^*_p},\ \underline{S}] \tag{9b}$$

$$\underline{C} = [\underline{0},\ 0,\ \underline{0},\ \underline{1}]$$

$$\underline{b} = [\underline{0},\ 0,\ \underline{W_D}]$$

$\underline{a}_i$ = i'th column of matrix $\underline{A}$

II. Determination of a Basic Feasible Solution

Eqs. (7) are a set of m linear equations in n unknowns ($n \geq m$). According to the LP theory, for the solution of the just stated LP problem it is required to know a basic feasible solution of Eqs. (7)

$$\underline{x}_B = \underline{B}^{-1} \underline{b} \tag{10}$$

and the vectors

$$\underline{y}_j = \underline{B}^{-1} \underline{a}_j \quad (j = m+1,n) \tag{11}$$

where $\underline{B}$ is a nonsingular matrix whose columns are any m linearly independent columns from $\underline{A}$ and $\underline{a}_j$ ($j = m+1,n$) are the columns of $\underline{A}$ out of $\underline{B}$.

A basic feasible solution and the vectors $\underline{y}_j$ can be obtained from relations (10) and (11) by inverting a basic matrix $\underline{B}$. If the size of $\underline{B}$ is large its inversion requires excessive computer time and gives rise to prohibitive roundoff errors. The size of $\underline{B}$ depends on the reactor geometry, the number of neutron groups, and on how the desired power distribution is defined.

To control the shape of the power density distribution a continuous poison distribution is required. In practical reactor designs the poison distribution (control rods, burnable poison) is not continuous. There is a finite number of control rods and a finite number of different burnable poison concentrations. By varying them the power density distribution cannot be matched point by point to a desired power density distribution. This difficulty is avoided if the reactor is divided into a number of regions R, and in each region a desired value is assigned either to the average power density or to the peak power density and as desired power

distribution is defined the distribution of these values. The number R must be smaller or equal to the number of control rods.

Even for a small number of regions, R, and a simple reactor geometry matrix $\underline{B}$ remains large. For example, for a 1-D reactor divided into four regions, one hundred mesh points and for five neutron groups matrix $\underline{B}$ is of the order of 500x500. The inversion of $\underline{B}$ can be avoided by noting that a basic feasible solution and the vectors $\underline{y}_j$ can be obtained from solutions of the multigroup diffusion equation as follows. For $\underline{\delta N}_p = \underline{0}$ Eqs. (5) are reduced to

$$\begin{aligned} \underline{L}\,\underline{\phi}^o + \underline{\beta}_\ell \cdot 0 &= \underline{0} \\ \text{Pert}_\ell \cdot 0 &= 0 \\ \underline{P}\,\underline{\phi}^o + \underline{E}\,\underline{S}^o &= \underline{W}_D \end{aligned} \tag{12}$$

The vector $\underline{\phi}^o$ is the solution of the multigroup diffusion equations with poison distribution $N_p^o(\bar{r})$ (the operating poison distribution). The components of the vector $\underline{S}^o$ are the difference between the components of the vectors $\underline{W}_D$ and $\underline{P}\,\underline{\phi}^o$. The elements of the matrix $\underline{E}$ are defined such that the vector $\underline{S}^o$ is a positive vector. As has already been mentioned, $\underline{\phi}^o$ is normalized such that one of the components of the vector $\underline{S}^o$ is equal to zero.

It is obvious that

$$X_B = [\underline{\phi}^o,\ 0,\ \underline{S}^o] \tag{13}$$

is a basic feasible solution of the optimization problem defined by

Eqs. (7) and (8) and corresponds to a basis matrix

$$\underline{B} = \begin{bmatrix} \underline{L} & \underline{\beta}_{\ell} & \underline{0} \\ 0 & \text{Pert}_{\ell} & 0 \\ \underline{P} & \underline{0} & \underline{E} \end{bmatrix} \tag{14}$$

III. Determination of the Vectors $\underline{y}_j$

The vectors $\underline{y}_j$ are solutions of the equations

$$\underline{B}\,\underline{y}_j = \underline{a}_j \tag{15}$$

where $\underline{a}_j$ are the columns of matrix $\underline{A}$ out of the basis matrix $\underline{B}$. Since $\underline{B}$ has been defined by Eq. (14), $\underline{a}_j$ is the j'th column of the matrix

$$\underline{T} = \begin{bmatrix} \underline{M} \\ \underline{\text{Pert}} \\ \underline{0} \end{bmatrix} \tag{16}$$

If the poison distribution is perturbed around the operating poison distribution $\underline{N}_p^o$ by

$$\underline{\delta N}_p = [\delta N_{p,\ell}, 0, \ldots, \delta N^*_{p,j}, 0, \ldots, 0] \tag{17}$$

where $\delta N^*_{p,j}$ is the j'th component of the vector $\underline{\delta N^*}_p$, and such that the reactor remains critical, Eqs. (5) are written

$$\begin{aligned} \underline{L}\,\underline{\phi} + \underline{\beta}_{\ell}\delta N_{p,\ell} + \underline{\beta}_j \delta N^*_{p,j} &= 0 \\ \text{Pert}_{\ell}\delta N_{p,\ell} + \text{Pert}_j \delta N^*_{p,j} &= 0 \\ \underline{P}\,\underline{\phi} + \underline{E}\,\underline{S} &= \underline{W}_D \end{aligned} \tag{18}$$

Subtraction of Eqs. (12) from Eqs. (18) gives

$$\underline{L}\,(\underline{\phi}-\underline{\phi}^{o}) + \underline{\beta}_{\ell}\delta N_{p,\ell} + \underline{\beta}_{j}\delta N^{*}_{p,j} = \underline{0}$$

$$\mathrm{Pert}_{\ell}\delta N_{p,\ell} + \mathrm{Pert}_{j}\delta N^{*}_{p,j} = 0$$

$$\underline{P}\,(\underline{\phi}-\underline{\phi}^{o}) + \underline{E}(\underline{S}-\underline{S}^{o}) = \underline{0}$$

or

$$\underline{L}\,\frac{\underline{\phi}-\underline{\phi}^{o}}{-\delta N^{*}_{p,j}} + \underline{\beta}_{\ell}\,\frac{\delta N_{p,\ell}}{-\delta N^{*}_{p,j}} = \underline{\beta}_{j}$$

$$\mathrm{Pert}_{\ell}\,\frac{\delta N_{p,\ell}}{-\delta N^{*}_{p,j}} = \mathrm{Pert}_{j} \qquad (19)$$

$$\underline{P}\,\frac{\underline{\phi}-\underline{\phi}^{o}}{-\delta N^{*}_{p,j}} + \underline{E}\,\frac{\underline{S}-\underline{S}^{o}}{-\delta N^{*}_{p,j}} = \underline{0}$$

But the coefficient matrix of (19) is $\underline{B}$ and $[\underline{\beta}_{j}, \mathrm{Pert}_{j}, \underline{0}]$ is the j'th column of matrix $\underline{T}$. Therefore

$$\underline{y}_{j} = \left[\frac{\underline{\phi}-\underline{\phi}^{o}}{-\delta N^{*}_{p,j}}\,,\ \frac{\delta N_{p,\ell}}{-\delta N^{*}_{p,j}}\,,\ \frac{\underline{S}-\underline{S}^{o}}{-\delta N^{*}_{p,j}}\right] \qquad (20)$$

In conclusion, a good approximation to the vector $\underline{y}_{j}$ is obtained as follows: The operating poison distribution is perturbed by

$$\underline{\delta N}_{p} = [\delta N_{p,\ell}, 0, \ldots, \delta N^{*}_{p,j}, 0, \ldots 0] \qquad (21)$$

where $\delta N^{*}_{p,j}$ is a small known quantity such that second order terms can be neglected and $\delta N_{p,\ell}$ is determined by perturbation theory (Eq. (4)). The multigroup diffusion equations are solved and the flux vector is normalized the same way as it was normalized in the determination of the basic feasible solution. Then $\underline{y}_{j}$ is determined by Eq. (20).

The determination of the basic feasible solution and the vectors $\underline{y}_j$ from standard solutions of the multigroup diffusion equations, renders to the method the advantage that it can be used for any reactor geometry. For complex reactor geometries the limitations are basically the same as for the solution of the multigroup diffusion equations. The inversion of the basis matrix $\underline{B}$ would limit the applicability of the method to simple geometries and very few neutron groups.

IV. Remarks

If I is the number of components of the vector $\underline{\delta N}^*_p$ (number of different poison concentrations -1), I solutions of the multigroup diffusion equations are required to determine the vectors $\underline{y}_j$ (j=1,I). Therefore the computation time required to calculate $\underline{y}_j$ increases linearly with I. Some computer time saving can be achieved if a good starting flux guess is available. Since $\delta N^*_{p,j}$ is a small perturbation, the flux vector $\underline{\phi}^o$ of the basic feasible solution could be such a guess. The numerical applications of the method show that the use of $\underline{\phi}^o$ as a starting flux saves considerable computation time.

Since the flux vector $\underline{\phi}$ is greater than zero the columns of the basis matrix $\underline{B}$ which correspond to the vector $\underline{\phi}$ cannot be removed out of basis. Therefore in each LP iteration the calculations to determine the column to be removed out of basis can be restricted only to the columns corresponding to the vectors $\underline{\delta N}_p$ and $\underline{S}$. This is a great computational advantage since the number of columns corresponding to the flux vector $\underline{\phi}$ is much greater than the number of columns corresponding to the vectors $\underline{\delta N}_p$ and $\underline{S}$. For example, at the 1-D problem mentioned

earlier there are 500 columns corresponding to the flux vector and only seven columns corresponding to the vectors $\underline{\delta N}_p$ and $\underline{S}$.

Since the components of the vector $\underline{\delta N}_p$ are unrestricted in sign, for the columns corresponding to the vector $\underline{\delta N}_p$ the LP algorithm for unrestricted variables is applied.[4] This does not impose any specially additional computation requirements. For the columns corresponding to the vector $\underline{S}$ the standard algorithm for positive variables is applied ($\underline{S}$ is $\geq \underline{0}$).

The treatment of constraints on the poison concentration of the form

$$q_j \leq \delta N_{p,j} \leq u_j$$

where q_j and u_j are known lower and upper limits, is straightforward and the same as in standard methods of LP routines.[4] Since the previous mathematical analysis does not add anything new with respect to them, in order to avoid notational complications, they have not been included in it.

NUMERICAL RESULTS

For the application of the developed method a computer code has been written and numerical results have been obtained for a fast reactor of infinite cylindrical geometry. The code is divided into two main parts. The first one solves the multigroup diffusion and multigroup adjoint equations to obtain the basic feasible solution and the vectors $\underline{y}_j$. The second part uses as input the basic feasible solution and the vectors $\underline{y}_j$ and does the optimization calculations. As has been mentioned the method can be used for any reactor geometry. For its numerical application to

a given geometry it is only required to use in the first part of the computer code routines which solve the multigroup diffusion and multigroup adjoint equations for the given geometry.

The reactor has been divided into four core regions. Two numerical tests have been performed. In the first one the poison distribution has been determined which results in a desired average power density distribution. In the second test the poison distribution has been determined which results in a desired peak power density distribution.

As poison has been used a fictitious material having an atomic density equal to the molecular density of UO_2 and absorption cross sections equal to twice the corresponding absorption cross sections of UO_2. The other poison cross sections have been taken as equal to zero.

For the neutronic calculations five neutron groups have been employed.

The reactor dimensions and composition are given in Tables I and II respectively. The sum of the Na and poison volume fractions has been constrained to be equal to 50%. This means that if poison is removed it is replaced by Na and if poison is added it replaces Na.

Table III shows the initial (operating) poison distribution, the initial power distribution and the desired power distribution. The poison distribution which results in the desired power distribution (optimum poison distribution) is shown in Table IV. Because of the approximations made in the development of the method, the power distribution which results from the computed optimum poison distribution does not agree exactly with the desired power distribution given as an input. The resultant power distribution is also shown in Table IV.

As it is shown in Table III, the ratio of the maximum average power density to the minimum average power density for the initial poison distribution is 2.237. The same ratio of the peak power densities is 2.136. The desired power distributions are much flatter than the initial distributions. The just mentioned ratios for the desired distributions are 1.373 and 1.232 respectively. They are by 38.6% and 42.3% smaller than their respective values in the initial power distributions.

Comparison of the desired and resultant power distributions (Table IV) shows the following. The ratio of the maximum average power density to the minimum average power density of the initial power distribution has been reduced to 1.439 or by 35.7% instead of the desired 38.6%. The same ratio of the peak power densities has been reduced to 1.243 or by 41.8% instead of the desired 42.3%.

Taking into account that second order terms in poison absorption (Eqs. (3)) have been neglected and that the desired distributions are considerably different than the initial distributions, the agreement between desired and resultant power distributions must be considered as very good. As it should be expected this agreement becomes better as the initial and desired distributions get closer. In Table V are shown the results obtained for a desired distribution closer to the initial distribution than in the previous cases. The ratio of the maximum average power density to the minimum average power density in the desired distribution is by 25.2% smaller than in the initial distribution. In the resultant distribution the same quantity is by 26.1% smaller than in the initial distribution. As it was expected, the agreement between resultant and desired distributions is better than in the previous case.

It must be pointed out that if there is a large difference between the initial and desired distributions, the solution can be improved by running the code twice where the output of the first run has been used as input of the second run.

The output of the LP algorithm in addition to the poison distribution which results in the desired power distribution gives also the corresponding flux distribution (see Eq. (9b)). The flux distribution obtained from the optimization part of the developed code has been compared with the flux distribution obtained from a direct solution of the multigroup diffusion equations where as input has been used the optimum poison distribution. They differ by less than $\sim$6% for the poison distributions of Table IV and by less than $\sim$3% for the poison distribution of Table V. The agreement in the second case is better because the initial and desired power distributions are closer than in the first case.

The vectors $\underline{y}_j$ have been obtained from solutions of the multigroup diffusion equations by applying small known perturbations on the operating poison distribution (Eqs. (20), (21)). The sensitivity of the optimum poison distribution to the magnitude of the perturbation $\delta N^*_{p,j}$ has been examined. Computations have been performed with $\delta N^*_{p,j}$ equal to 0.01, 0.02, 0.03. The obtained optimum poison distributions are practically the same.

The computer time for the calculations is equal to the time required for: one solution of the multigroup diffusion equations to obtain the basic feasible solution (Eqs. (12)), one solution of the adjoint multigroup diffusion equations to obtain the vector [$Pert_\ell$, $\underline{Pert}$] (Eqs. (5)), I (number of different poison concentrations -1) solutions of the multigroup

diffusion equations to obtain the vectors $\underline{y}_j$, plus the time required for the LP iterations. As has been already mentioned, computer time can be saved if as a starting flux for the calculation of the vectors $\underline{y}_j$ the flux vector $\underline{\phi}^o$ of the basic feasible solution is used. In the discussed numerical applications I is equal to three, $\underline{\phi}^o$ has been used as a starting flux for the calculation of $\underline{y}_j$, while for the calculation of $\underline{\phi}^o$ and the solution of the adjoint equations a flat starting flux has been used. The computation time for the vectors $\underline{y}_j$ and the LP iterations on a CDC 3300 is slightly less than the time required to calculate $\underline{\phi}^o$. Since the computer time for the solution of the adjoint equations is about equal to the time required to obtain $\underline{\phi}^o$, the total computation time is about equal to three times the computation time required to obtain $\underline{\phi}^o$ by solving the multigroup diffusion equations.

SUMMARY AND CONCLUSIONS

The problem of determination of the poison distribution in a reactor which results in a desired power distribution is formulated and solved as a linear programming problem. For realistic reactor problems (two or three dimensions, many neutron groups) the resulting LP problem is very large and its solution by known LP routines requires excessive computer time and gives rise to prohibitive roundoff errors. To avoid this difficulty, all the input required to start the LP algorithm is not calculated by the known methods of the LP routines but from solutions of the neutronic equations. By this way the computational problems are practically eliminated to those of the methods used to solve the neutronic equations.

The output of the LP algorithm in addition to the poison distribution gives the corresponding neutron flux distribution. Thus if the method is

incorporated in a depletion code, it determines: a) the poison distribution which retains the reactor critical and results in the desired power distribution, and b) the neutron flux for the depletion calculations of the next step.

The developed method can be also used to determine the distributions of any materials in the reactor which result in a desired power distribution or maximize or minimize reactor parameters which either are or after linearization become linear functions of the neutron flux and the material concentrations.

The present method can form the basis of further work towards the development of an automatic control system where a computer using data from the reactor determines the control rod patterns and feeds them directly to the control rod adjustment system of the reactor.

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Table I. Reactor Dimensions

Region		Inner Radius (cm)	Outer Radius (cm)
Core	1	0.00	55.68
	2	55.68	80.04
	3	80.04	97.44
	4	97.44	111.36
Radial Blanket	5	111.36	156.60

Table II. Reactor Composition

Material	Core Regions (vol %)				Blanket	Atomic or Molecular Density for Pure Materials (10^{24} cm^{-3})
	1	2	3	4		
PuO_2	3.829	3.927	4.736	5.775	--	0.025189
UO_2	31.171	31.073	30.264	29.225	35	0.024444
Fe	15				15	0.084870
Na	50				50	0.025410
Poison					--	0.024444

Table III. Initial Poison and Power Distributions and Desired Power Distribution

Region	Poison (vol %)	Relative Average Power Density		Relative Peak Power Density	
		Initial	Desired	Initial	Desired
1	1.670	2.237	1.373	2.136	1.232
2	7.226	1.542	1.287	1.493	1.141
3	7.226	1.287	1.210	1.215	1.122
4	12.049	1.000	1.000	1.000	1.000

Table IV. Optimum Poison Distributions, Resultant Power Distributions and Desired Power Distributions

Region	Poison (vol %)	Relative Average Power Density		Poison (vol %)	Relative Peak Power Density	
		Resultant	Desired		Resultant	Desired
1	5.020	1.439	1.373	4.735	1.243	1.232
2	4.011	1.295	1.287	4.779	1.126	1.141
3	6.178	1.209	1.210	5.297	1.113	1.122
4	6.551	1.000	1.000	6.612	1.000	1.000

Table V. Optimum Poison Distribution, Resultant and Desired Power Distributions for a Desired Distribution Closer to the Initial Distribution Than in the Case of Previous Table

Region	Poison (vol %)	Relative Average Power Density	
		Resultant	Desired
1	2.819	1.653	1.673
2	6.531	1.266	1.287
3	9.257	1.150	1.163
4	2.377	1.000	1.000